Note: This test bank was developed by Joshua Stangle (University of Wisconsin – Superior) to accompany the fourth edition of *Fundamentals of Probability: With Stochastic Processes* by Saeed Ghahramani.

**Chapter 1**

(1) The waiting time (in seconds) between one song’s end and the next song’s beginning on a certain radio station is a random number between 3 and 6.8. Find the probability that the time between one song and another is at least 4.9 seconds.

Answer:

(2) Two fair 20-sided dice are rolled. How many total possible outcomes are there? What is the probability that the second number rolled is exactly 4 more than the first?

Answer: There are 400 total possible outcomes. The probability the second is 4 more than the first is 16/400.

(3) At the Internal Revenue Service, a social security number (9-digits) is selected randomly to receive an audit. (a) What is the event that the last two digits are odd? (b) What is the event that the last two digits form a number divisible by 5?

Answer: (a) .

(b) .

(4) Let . Choose two distinct 2-element subsets at random from , the set of all subsets of . Find the size of the sample space of this experiment. Describe the following events explicitly:

Sample Space: Let denote the sample space, then .

(a) The two sets are disjoint.

Answer:

(b) The intersection of the sets is .

Answer:

(c) The complement of the union of the sets consists of only the element .

Answer:

(5) At a given company there are 4 employees: Jim, Pam Dwight, and Michael. There is a breakroom and each employee can be inside or outside of the breakroom at any given time. Let denote the event that Jim, Pam, Dwight, and Michael are in the breakroom, respectively. In terms of the events , and describe the event that at most two people are in the breakroom at a given time. What about the event that at least three people are in the breakroom?

Answer: The event that at most two people are in the room at the same time is

The event that at least 3 people are in the breakroom is

(6) George likes antiques. He purchases an antique chair for $200 and plans to refurbish it. Give an explicit sample space for the resale value of his chair after he refurbishes it. Define**,** in set notation**,** the event that he loses money on his chair. Suppose George will only charge whole dollar amounts for convenience.

Answer: The sample space for the value of his chair is . The event he loses money is .

(7) Suppose we have a sample space and two events and . Suppose . Which statement is false?Show a counterexample for your answer**.**

(a) cannot occur unless has occurred.

(b) cannot occur unless has occurred.

Answer: (a) Is false. Let , and . Then if the experiment turns up 4, has occurred without occurring.

(8) At a certain ice cream parlor, 40% of patrons get hot fudge, 25% do not get whipped cream, and 30% of patrons get both hot fudge and whipped cream. How many get hot fudge or whipped cream.

Answer: 35%

(9) A fair (6-sided) die is rolled until the first time two even numbers are rolled in a row. Give 10 elements of the sample space.

Answer: The following is an example: let each tuple represent the result of the rolls beginning at roll one. Then, ten elements of the sample space are

(10)A lion, who hunts only gazelle, buffalo, and giraffe**,** wakes up and hunts her first meal. If she is twice as likely to hunt a gazelle as a buffalo, and three times as likely to hunt a buffalo as a giraffe, find the respective probabilities of the lion hunting a gazelle, buffalo, and giraffe.

Answer: Let denote the events that the lion hunts a gazelle, buffalo, and giraffe, respectively. Then , and .

(11) In order to be crowned “Best in Town,” John**,** a tennis player from Newton, MA must beat both Melissa and Jeffrey at tennis. The probability that he beats Melissa is 50% and the probability he beats Jeffrey is 61%. If the probability that he beats at least one of them is 93%, find the probability that John is crowned “Best in Town.”

Answer: 18%

(12) Let be the sample space of some experiment. Let be given by . Can we define a probability distribution on so that ?

Answer: No, since for a probability distribution we need and

(13) Jeff has 3 pens**,** which he uses to do professional illustrations. He has a red pen, a blue pen, and a green pen. He often works on illustrations outside of home, and it is important that all pens are at least half-full with ink. Suppose a pen can be full, half-full or empty and that the level of ink in a pen is independent of the level of ink in other pens. How large is the sample space of this experiment? Describe the event that all three pens are at least half-full.

Answer: The sample space contains 27 elements. The event that all three pens are at least half-full is

(14) **F**or two events and explain why the following are impossible:

Answer: (1) is impossible because , therefore . (2) is impossible because therefore .

(15) A certain auto shop receives 7 autos, each on a random day, in the next seven days. The Smiths drop off all three of their vehicles while the Jones and the Masoods each drop off two vehicles. Write a sample space for the number of days between the day the Smiths drop off a second vehicle and the day they drop off a third vehicle (for instance if they drop off their second vehicle on the third day and their third on the fourth day, one day is between these drop off times.) From this sample space, describe the event that the Jones drop off their two vehicles on the last two days of this period.

Answer: The sample space is given by . The event the Jones drop their vehicles off on the last two days is .

**Chapter 2**

(1) An urn contains balls numbered one through 12. A person selects five balls at random and without replacement. What is the probability that they form a consecutive sequence of balls?

Answer:

(2) Three fair twenty-sided die are rolled (these have faces labelled 1-20). What is the probability they each show a prime number after rolled?

Answer:

(3) A hostess wants to seat a party of 9 around a circular table. John and Jennifer cannot sit next to each other. How many ways can she arrange the table?

Answer: 6\*(7!)=30240

(4) 15 students in Mrs. Studebacher’s class will be arranged randomly into three rows of 5 for a class picture. What is the probability the tallest 5 students occupy the top row? (Assume all the students are distinct heights.)

Answer:

(5) A deli has 15 turkey and 7 ham sandwiches wrapped in aluminum foil unmarked. If they select 5 sandwiches at random, what is the probability they are all turkey?

Answer: .

(6) A certain band consists of 20 players all of whom can play any instrument. They want to divide the players into a trio (group of 3), three quartets (group of 4), and a quintet (group of 5). How many ways can they do this?

Answer:

(7) In a Bridge game, each of the 4 players is dealt 13 cards from a standard deck of cards at random. What is the probability two of the four players each get exactly two aces?

Answer: .

(8) You play a game against a computer, which guesses your favorite Marvel comic book character. You answer a series of 15 yes or no questions, and the computer guesses your character based on the answers. If the computer is always correct, and for each sequence of yes’s and no’s there is a character, how many possible distinct characters are there?

Answer:

(9) Gregory is new to a city and is speed dating. He can choose to go or not to go on a date with any number of 14 different suitors. If he goes on a date, he takes his date either to dinner or to see a movie, but not both. How many different options does Gregory have?

Answer:

(10) A valet who is completely color-blind is rearranging the cars of a parking lot. He has 5 Honda Civics, 4 Ford Fusions, 3 Land Rover LR1s and 3 Mercedes E500s. If the cars of the same model are indistinguishable how many ways can the valet park these in 3 rows of 5?

Answer:

(11) Prove the following binomial relationship:

Hint: Consider an urn containing red balls and white balls. Consider the number of ways to select balls from this urn at one time.

Answer: For the left-hand side this is obviously the number of ways to select balls from an urn containing . For the right-hand side we consider the case we select red balls for (which means we must select white balls. We can do this in ways. To account for all values of we sum over , which is exactly the quantity on the right-hand side.

(12) For a certain setlist, a band wants to pick 3 songs from each of their 3 albums. If one of the albums contains 10 songs, one contains 11 songs, and one contains 12 songs, how many possible setlists are possible?

Answer: .

(13) A company makes 50 deposits to their bank in one month. Four of the deposits are mistakenly written in the company’s ledger as $2 more than actually deposited and three of the deposits are mistakenly recorded in the ledger as $2 less than was actually deposited. If the CEO checks the book keeping by comparing the sum of 6 random deposit receipts to the sum of their records in the ledger, what is the probability he thinks the ledger is accurate?

Answer: .

(14) A soccer team is made of 14 players. The game requires a structure of 4 forwards, 3 midfielders, 5 defenders, and 2 goal protectors. If each player can be assigned to any of the positions, how many ways can the players be assigned to these positions?

Answer: .

(15) Suppose that Janet bets on 12 different horse races and gets two correct. If she loses 7 of her tickets, what is the probability the remaining 5 have at least one winning ticket?

Answer: .

**Chapter 3**

(1) Two friends are running a 5K together. If their finishing times are random in the range of 26 to 40 minutes, and they finish independently of one another, what is the probability that one finishes in under 32 minutes, while the other takes at least 37 minutes?

Answer: 2.

(2) A machine produces batches of 10 light-bulbs at a time. The probability any given light bulb will be defective is 15%. A batch is rejected if at least 3 light-bulbs are defective. Find the probability a given batch is rejected.

Answer:

(3) Your cell phone battery is getting quite low, and you know that it will turn off randomly between now and the next 50 minutes. You are talking with a dear friend. If you manag~~e~~ to talk for 30 minutes, what is the probability that your phone lasts at least 12 more minutes?

Answer:

(4) You have four $1 bills, three $5 bills, and a $10 bill. Your friend Abdullah has seven $1s, two $5s, and three $10s. You notice that one of you has dropped a bill on the ground. If it is a $5, what is the probability that it was your bill?

Answer:

(5) Suppose that3 fair four-sided dice are rolled, and we’re told the sum of these is even. What is probability one of the rolls was a 3?

Answer:

(6) For lunch, every day, two of your friends, Mary and Jenn either have pizza or salad. Mary eats pizza 5 days per week and salad twice. Jenn eats pizza once a week and salad 6 times. Referring to Mary and Jenn, on a certain day, you are told that one of your friends had pizza, but they cannot tell you which friend. What is the probability that it was Mary who was seen having pizza?

Answer:

(7) You and your friend alternate drawing cards from a standard deck at random with replacement. The game ends when either you draw a red card, or he draws a face card (Jack, Queen, King). You draw first. What is the probability that you win (i.e., you draw a heart before he draws a face card)? You may leave your answer as an infinite sum.

Answer:

(8) In genetics, blonde hair (B) is dominant over red hair (r). Gary has blonde hair, meaning he either has genetic pairing BB or Br. He is unsure of the probability of each outcome. His doctor tells him if he marries a woman with red hair (genetic marker rr) their child will be red haired with probability 3/8. What is the probability that Gary has genetic marker BB?

Answer: .

(9) Construct a sample space and two events and so that .

Answer: Let with all outcomes equally likely. Let and

(10) A box contains 15 quarters, one of which has heads on both sides. The remaining 14 are normal. You pick a quarter out of the box at random and flip it six times. All six times you get heads. What is the probability it is the 2-headed quarter?

Answer:

(11) At a certain radio station, they receive requests all day via email. They play a mix of requested songs and songs chosen by the DJ; they always play 21% of songs requested by 9am, 31% of the songs requested between 9am and 2pm, and 8% of songs requested after 2pm. If in a given day 14% of requests are received by 9am and 43% are received by 2pm (including those by 9am), what percentage of songs played that day are chosen by the DJ?

Answer: .

(12) A baseball team has 4 pitchers and 4 catchers, all of whom play in a given game. The team manager records a defensive success if at least 2 of the pitchers and 3 of the catchers make no mistakes. In a given game, a pitcher makes no mistakes with probability .6 independently of any other player and a catcher makes no mistakes with probability .7. Give an exact expression for the probability that a randomly observed game is recorded as a success.

Answer:

(13) A certain type of dog can have long hair, medium hair, or short hair. For any length of hair, the dog can be white, grey, yellow, or brindle. Let be the events oflong hair, medium hair, and short hair, respectively. Let be the events that a dog is white, grey, yellow, or brindle respectively. Denote for . Give a partition of this breed of dogs into 6 sets.

Answer:

(14) Urn I contains 3 black balls, 2 white balls, and a red ball. Urn II contains 5 black balls, 3 white balls, and 6 red balls. Urn III contains 4 black balls, 3 white balls, and 3 red balls. An urn is selected at random, one of the balls is removed randomly, observed to be white and not replaced. If another ball is then selected at random from the same urn, what is the probability it is black?

Answer: .

(15) At a certain stream, there are salmon and there are trout. Javon, Smitty, and Alice caught 22 fish and mixed them together; they have 14 salmon and 8 trout. Now, they can’t remember who caught what. Javon knows that he caught 8 fish and the first 4 were trout. What is the probability that at least 2 of the last 4 he caught were Salmon?

Answer:

**Chapter 4**

(1) You pick a card at random and with replacement twice from 3 cards numbered 1, 2, 3. Let S denote their sum and D the result of the first roll minus the second roll. Give the probability mass function of the product of S and D.

Answer:

(2) Let each letter of the alphabet have numerical value equal to its position; i.e., A=1, B=2, , Z=26. A 3-letter word is constructed at random from the letters A, B, C, D, E (any order of letters counts, nonsense words are acceptable, letters cannot be repeated). You then add the numerical values of the letters. Find the expected value of the word.

Answer: 9

(3) Of US citizens, approximately 12% have traveled internationally. A current company wants to hire a new employee who has traveled internationally. How many applicants do they need to interview to have a 60% chance that at least one of the applicants has traveled internationally?

Answer: 8

(4) There are 10 different accounts under study at a local credit union. 3 have $10,000 in the account, 2 have $12,500, 4 have $15,000, and 1 has $20,000. An account is selected at random. Find the expected value and the variance of the money inthe account.

Answer: $13,500

(5) Consider the following function:

(a) Find the value of so that is a probability density function. (b) Let be a random variable with this probability density function. Find the probability that is between 1 and 1.5. (c) Find the probability that the function is increasing.

Answer: (a) (b) .254 (c) 15/16.

(6) Giant squids have one offspring at a time until they have a male offspring, at which point they quit reproducing. If they never have a male, they stop after 4 female offspring. They have male offspring with probability .25. Let be the number of female offspring for a giant squid. Find the probability mass function and the distribution function of .

Answer: 2.7345

(7) Let be a discrete random variable with probability mass function:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | -1 | 1 | 2 | 3 |
| p(x) | .2 | .3 | .15 | .35 |

Find

Answer: 10.75

(8) Busses arrive at a certain stop such that in a time interval of length the number of arrivals is a random variable with probability mass function

(a) Prove that is indeed a probability mass function.

(b) Find the probability of at least 3 arrivals in one time period of length .

Answer: (a) (b)

(9) A drawer contains 12 pairs of long socks and 10 pairs of short socks. If a traveler selects 4 pairs of socks at random to pack, what is the probability mass function of the number of pairs of long socks?

Answer:

(10) Suppose that is a random variable such that , . Find .

Answer: .

(11) The number of finishers at a particularly difficult race is a random variable with probability mass function:

Find the probability that at least 3 people do not finish.

Answer:

(12) Suppose you possess a loaded 6-sided die in which 1 is twice as likely as each of the other 5 outcomes. Let be the outcome of rolling this die once. Find .

Answer: .

(13) On a certain athletics team, athletes get a $250 bonus if they run their one-mile race in under 4 minutes. If they do not, they pay the club $10. There are 7 athletes on the team. The number of athletes who run their mile in under 4 minutes has the probability mass function.

The bonuses are withdrawn from and the fines are charged to the team’s account. After all the athletes have run their mile, what is the expected value of the change in the balance of the team’s account?

Answer:

(15) Let be the number of cars belonging to a random family in Beverly Hills. Suppose that has distribution function where , , , , , also suppose . Find for .

Answer: , , , , , .

**Chapter 5**

(1) Suppose that a certain car manufacturer makes 10 cars on some day; 4 being red and 6 being blue. A car dealer picks 5 cars at random from the 10 and purchases them. What is the probability that she gets at least 2 red cars?

Answer:

(2) John claims that he is mildly psychic and can guess the color of a card withdrawn from an ordinary deck of cards correctly 70% of the time. His friend selects cards at random from the deck (replacing them and shuffling each time). He selects 12 cards and John guesses 9 correctly. (a) Find the probability that John guesses exactly 9 correctly if his claim of being psychic is true. (b) Find the probability thathe gets exactly 9 correct if he is just guessing.

Answer: (a) (b) .

(3) People are in line for a free T-shirt. The company knows that 30% of people select a red shirt, 45% select a grey shirt, and 25% select a black shirt. Of the next 5 people who choose a shirt, what is the expected number of people who select a red shirt?

Answer: Expected Value is 5(.3)=1.5.

(4) You select cards randomly until you get the queen of spades, replacing the card and shuffling each time. What is the probability that it takes exactly 8 draws?

Answer:

(5) Assume that busses arrive at a stop randomly and at a Poisson rate of 4 per hour. If they begin arriving at 9am, what is the probability that 5 busses arrive by 10:30 am?

Answer: .

(6) A batter gets a hit in 35% of her at-bats, independently of other at-bats. If is the number of at-bats she must attempt to get 1 hit, what is the variance of and ?

Answer: .,

(7) Your friends went apple picking and gave you 10 apples from their stash. If their stash contained 50 good apples and 12 rotten apples, what is the probability that your share contained at least 2 rotten apples?

Answer: .

(8) Suppose that a certain book club has 12 members. Five of the members like romance and seven like mystery. If you select 4 members at random, what is the expected value of the members in the sample who like romance?

Answer:

(9) In a large city, a radio station gets calls all day at a Poisson rate of 5 per hour. If they are giving out 20 tickets to the first 20 callers, what is the probability that they give away all the tickets in exactly 3 hours?

Answer: .

(10) In a certain community, 80% of residents hand out Halloween candies and 20% do not. If you visit 12 houses randomly, what is the probability that you get candy from at least 9 of them?

Answer:

(11) Suppose that a low end dry cleaner has a 5% chance of losing any particular garment, independently of the other garments. You drop off12 garments at this dry cleaner. What is the probability that you get at least 10 of them back?

Answer: .

(12) For not forfeiting a match, a university intramural team must have 3 female competitors and 3 male competitors. A certain team has 12 female members and 9 male members. If the female members show up for a match independently of one another with probability .25, what is the probability that at least 3 females arrive to the match.

Answer: .

(13) You walk around a large city asking people at random whether they are born in the same month as you were, but you stop doing this if you ask 25 people without anyone saying yes. Let be the number of people you ask before someone says yes. (Assume that birthdays are equally distributed among the months.) Find and .

Answer: . .

(14) On a random day, of the dogs that are dropped off at a kennel**,** 45% are male, and of the male dogs, 33% are working breeds. Let be the number of the first 8 dogs arriving at the kennel on acertain day and are male and a working breed. Find the probability mass function of its expected value and its variance**.**

Answer: , , .

(15) In a small New England town, a non-lethal virus comes through, and 1 in 100 people who contract the virus show symptoms. Find the probability that the third person to show symptoms is within the first 400 people.

Answer: 0

**Chapter 6**

(1) Let be a random variable with uniform density function on . Find the density function of .

Answer: has density on and otherwise. For , .

(2) Suppose is a non-negative random variable with probability density function

Find .

Answer: .

(3) Find the constant so that

is a probability density function.

Answer: .

(4) Let be a continuous random variable with probability density function

Find the probability density function of .

Answer: .

(5) A lightbulb company knows that its lightbulbs have a lifespan (in thousands of hours) modelled by a random variable with the probability density function .

(a) Find

(b) If a certain company installs 3 of their lightbulbs at the same time, what is the probability that all three need to be replaced within 3500 hours?

Answer: (a) (b) .

(6) Let be a random variable with probability density for . Find .

Answer: . Note .

(7) Let be a random point in the interal . Find the probability density function of .

Answer: for .

(8) A dentist has found that a certain procedure has a cost (in thousands of dollars) with probability density function: . The dentist also found that if she charges the patients 10% of the cost up front, then *k*, the amount of money the insurance company pays, only in 12% of the time exceeds the remaining cost. Find .

Answer: , so so thousands of dollars (i.e., .)

(9) Let be a continuous random variable taking values in with distribution function and probability density function . Find the distribution and probability density functions of .

Answer: and .

(10) Suppose that the amount of water, in thousands of gallons, which fall on a city during arain storm has distribution function:

Find the probability that, in a rain storm,at least 2000 gallons of water fall, given that 1200 gallons have already fallen. Is this different than the probability that 800 gallons fall without any prior knowledge?

Answer: . This is the same as the probability 800 gallons fall with no other knowledge. The exponential density is memoryless.

(11) Assume thata certain dog sled team has race times, in hours, which are a random variable with probability density function

(a) Find the expected duration of such a race. (b) Find the probability that if the dogs run 5 races, then none of them is shorter than 3 hours. Assume the duration of the individual races are independent of one another.

Answer: (a)  (b)

(12) Let be uniformly distributed on . Find the probability density function and distribution function of .

Answer: The distribution function is

The probability density function is

(13) On a certain stretch of highway, to be pulled over by police, the speed at which a car must travel over the speed limit of 60mph is a random variable which has the following probability density function:

(a) Find the maximum speed at which a vehicle can go and be 90% sure that it will not be pulled over.

(b) Suppose Mary-Sue gets pulled over on this highway. What was the expected speed she was driving?

Answer: (a) mph.  (b) mph.

(14) Let be a continuous random variable with probability density function

Using the method of transformations, find the probability density function of .

Answer:

(15) Suppose that all students of a class of 12get tested for fever. Furthermore, suppose that the temperature of a student is a random variable with probability density function

If 7 or more students have a fever above , the class is dismissed. If the students’ temperatures are independent of one another, find the probability that they are dismissed.

Answer: 0.

**Chapter 7**

(1) Let be a random number selected from the interval . Find the expected value of .

Answer:

(2) At a certain university, there are exactly 3000 male undergraduates, 3000 female undergraduates, and 3000 gender non-binary undergraduates. If a group of 50 undergraduates is formed at random, what is an approximation of the probability that at least 20 of the members are female?

Answer:

(3) For a certain model of oven, the temperature of the oven when set to is a uniform random variable over the interval . Find the proportion of such ovens thatrun hotter than the indicated temperature.

Answer: .

(4) For a customer, the time spent at a help desk with an associate is exponential with mean 120 minutes. If 5 customers are at the help desk being helped (independently) by five associates, find the probability that the waiting times of at least 2 of them are less that 120 minutes.

Answer: .

(5) Suppose that incomes in a certain area are normally distributed with mean and standard deviation . Give the probability density function for the income of a randomly selected citizen in this region.

Answer: .

(6) Suppose that is an exponential random variable with parameter . Find .

Answer: .

(7) Suppose that in a 45-mph zone, the speeds at which drivers are pulled over are normally distributed with mean 60 mph and variance 25 mph. What is the maximum speed a driver can go with at most a 10% probability of being pulled over?

Answer: 48.4 mph

(8) Let be a uniform random variable over . Find the expected value of the random variable .

Answer: .

(19) At a concert, the fraction of attendees who dance is a beta randomvariable with parameters (4,3). Find the probability that out of 1000 concert attendees, at least 750 dance.

Answer:

(11) Every week, a station plays 310 blues songs and 620 jazz songs. If one day, a random block of 42 songs is played, what is the probability that at least 19 jazz and at least 19 blues songs are played?

Answer:

(12) When ordering pizza from a certain pizza parlor, the delivery time is an exponential random variable with mean 25 minutes. If **it** takes more than 30 minutes for the pizza to be delivered, they provide it for free. If it takes more than 40 minutes, not only will the customer get the pizza for free, but he or she will receive an additional $10. If a pizza with delivery costs $18, on average, how much does the parlor make per pizza delivery?

Answer: .

(13) A certain school is having a graduation ceremony, and graduates are directed onto the stage to receive their diplomas. If the school has graduates and they are called on stage at a Poisson rate of per minute, what is the expected value and standard deviation of the time until all of the graduates are called on stage?

Answer: minutes or hours, and the standard deviation is minutes.

(14) Suppose that a certain can is manufactured at a factory in Dayton, OH. The radius of the can is a normal random variable with mean .2cm and Variance .25cm. Find the probability distribution function of the radius of the can.

Answer:

(15) Suppose that the net worth of a member of an audience at a certain self-help seminar is a normal random variable with mean and standard deviation . Find the probability that in a group of 14 randomly selected audience members, at least 5 have net worth between and .

Answer: .

**Chapter 8**

(1) A certain aircraft can only fly if both of its two engines are functioning properly. This plane is a home project and so has two different engines. The lifetimes of the engines, in years, are given by random variable and and the joint probability density function of and is given by

(a) Find the probability that the airplane is capable of flying for more than 1 year.

(b) Find the marginal distribution function of .

Answer: (a)  (b)

(2) Let and Y be uniform random variables over (0,2) and (0,4), respectively. Find the probability that .

Answer: .

(3) To invite to a dinner,the White House randomly selects 9 athletes from a group of 5 runners, 7 gymnasts, and 6 boxers. Let be the number of gymnasts invited, and the number of boxers invited. Find the joint probability mass function of and and the conditional probability density function of given .

Answer: (a)

(b)

(4) Suppose that five numbers are chosen from 1 to 60 (inclusive, without replacement). If is the number of even numbers, and is the number of powers of 3 in the list of the five chosen, find the joint probability mass function of and .

Answer:

(6) A game is played by throwing a bean bag onto a circular game board of radius 3m. There is a region, a .5m-by-.5m square, in the center of the board. If the bag lands at a random location on the board, find the probability that it lands on the square.

Answer: .

(7) A point is selected at random in the square . Let and represent the and coordinates of the point, respectively. What is probability of the following event**s**, (a) (b) ?

Answer: (a) . (b) .

(8) Let and be two random variables with joint probability density function

(a) Find A.

(b) Find the marginal distribution function of .

Answer: (a) . (b) .

(9) A die is rolled successively. Let represent the number of 1s in the first 8 rolls and the number of 1s in the next 12 rolls. Find the joint probability mass function of and

Answer:

(10) Two numbers are picked in succession from 1 to 4 (inclusive, with replacement). Let denote the minimum of the numbers and the result of the first number minus the second. (a) Give the joint probability mass function of and . (b) Give the marginal probability mass function of and (c) find .

Answer: (a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -3 | 1/16 | 0 | 0 | 0 |
| -2 | 1/16 | 1/16 | 0 | 0 |
| -1 | 1/16 | 1/16 | 1/16 | 0 |
| 0 | 1/16 | 1/16 | 1/16 | 1/16 |
| 1 | 1/16 | 1/16 | 1/16 | 0 |
| 2 | 1/16 | 1/16 | 0 | 0 |
| 3 | 1/16 | 0 | 0 | 0 |

(b) .

(c)

(11) Suppose that weights of batteries of brand A are normally distributed with mean 35 grams and standard deviation 3 grams. Suppose thatweights of brand B batteries are normally distributed with mean 38 grams and standard deviation 4 grams. Find the probability that a randomly selected battery of brand A is heavier than a randomly selected battery of brand B.

Answer: 0.

(12) A point is selected at random from the trapezoid bounded by the lines , , and . Let be the -coordinate and the -coordinate of the point. Find . Are and independent?

Answer: . They are not independent since and .

(13) Let and have joint probability density function

Find the marginal probability density functions of and and

.

Answer: for .

. .

(14) In an experiment a standard die is rolled twice. Let be the sum of the rolls and be the first rollminus the second. Show that , but *X* and *Y* are not independent.

Answer: Let be the outcome of the first roll and the second. Then and . Then . Since and are identically distributed, it is clear that and . But, , while Thus, *X* and *Y* cannot be independent.

(15) Let and be independent random variables uniformly distributed over (0,1). Find the distribution function and probability density function of .

Answer: The distribution function is

The probabilitydensity function is

**Chapter 9**

(1) Let , , and be random variables with joint probability density function

Find .

Answer: .

(2) Three radio stations decide to play the same song independently of one another. They each start playing the song randomly sometime between 12:00pm and 12:25pm. (a) Find the probability that the song plays for the last time between 12:18pm and 12:22pm. (b) Find the probability that all three are played between 12:05pm and 12:20pm.

Answer: (a) (b) .

(3) You have two bank accounts, and you do not know what the balance of each account is. All you remember is that each account’s balance is a random amount between and . Find the expected value of the difference between the balances of the two accounts.

Answer: .

(4) Suppose that 70% of a certain car manufacturer’s production are sedans, 20% are SUVs, and 10% are trucks. If 17 vehicles of this manufacturer are selected randomly, what is the probability of exactly 9 sedans and exactly 1 truck or exactly 8 sedans and exactly 2 trucks?

Answer:

(5) The random variable , and have joint probability density function

(a) Find . (b) Find .

Answer: (a) (b) .

(6) A drawer contains 10 white socks, 14 blue socks, and 8 green socks. Suppose that 9 socks are chosen at random from the drawer, let be the number of white socks, the number of blue socks, and the number of green socks chosen. Give the joint probability mass function for , , and .

Answer:

(7) A random integer from 1 to 5 (inclusive) is chosen at random 12 times (with replacement). What is the probability that 1, 2, 4 and 5 appear 3 times each?

Answer:

(8) A certain sandwich shop has a cooler full of sandwiches. Suppose that45% of its sandwiches are turkey, 25% are ham, and 30% are vegetable sandwiches. A customer comes in and grabs 14 sandwiches randomly. What is the probability that she gets exactly 7 turkey and at most 3 vegetable sandwiches?

Answer: .

(9) A restaurant is about to close, but it must wait until all 8 remaining customers have left. Starting now, each customer’s remaining time in the restaurant is exponentially distributed with mean fifteen minutes. If the customers leave independently of one another, find the probability that (a) after 2 hours the restaurant is still open; (b) after 1 hour, the restaurant is still waiting on exactly 2 customers.

Answer:

(a)

(b)

(10) At a certain Kennel, 25% of the dogs admitted are Spaniels, %30 are Labradors, %24 are Collies, and the remainder are Mastiffs. If 25 dogs are randomly selected and let out to play, find the probability that 10 are Labradors, 4 are Collies, and at most 2 are Mastiffs.

Answer:

(11) You are making a Jello mold in the shape of an inverted right cone, radius 3cm at the bottom and 6 cm high. You place 3 cherries in the Jello for your guests to find. They end up distributed randomly in the Jello. Find the probability thatnone of the Cherries end up within 1cm of the exterior surface of the cone.

Answer:

(12) Suppose that you choose a point at random in the tetrahedron formed by the planes , , , and . Let represent the , , and -coordinates of that point, respectively. (a) Find the joint probability density function of , and . (b) Find the probability density function of marginalized over and .

Answer: (a) . (b) .

(13) For what value of is

a joint probability density function? For that value of , find .

Answer: and .

(14) A certain pool is staffed by unreliable lifeguards. The lifeguards are numbered 1 through and they all start on duty at the same time every day. The pool remains open as long as at least two of the lifeguards are on duty. Suppose that, on any day, independently of the others, lifeguard *i* stops working at a time which is a random variable with distribution function .In terms of the find the survival function of the period that the pool stays open.

Answer: where

**Chapter 10**

(1) A custodian has 9 identical keys of which only one opens room number 102’s door. However, she has forgotten which one. If she tries them randomly one at a time and without replacement, find the expected number of keys she must try before getting the correct one.

Answer: 5

(2) You roll a die 5 times. Let

For find .

Answer: .

(3) Let be randomly distributed with probability density function

Find .

Answer:

(4) Suppose that, and are random variables with standard deviations , and and that , and . Find.

Answer: .

(5) Two students are walking their dogs around a park. Mary’s dog sniffs a tree at a Poisson rate of 4 times per hour, while Antoine’s dog sniffs a tree at a Poisson rate of 3 times per hour independently of Mary’s dog. If Mary’s dog sniffed the last 2 trees, what is the probability that Antoine’s dog will sniff a tree next?

Answer: .

(6) A random point is chosen at random from the rectangle . Find .

Answer: .

(7) Let and have joint probability density function

Find .

Answer:

(8) You are rolling a fair 20-sided die. Find the expected number of rolls you need to toss to get 3 consecutive rolls of outcomes 13 or above**.**

Answer: .

(9) A dating app is meant to arrange dates between 13 dog lovers and 13 dog owners, each dog lover is matched with one dog owner. However, due to a glitch, instead of sending dog lovers to the matched dog owners,the app sends the dog lovers to the houses of dog owners randomly. Find the expected number of dog lovers who end up at the correct home.

Answer: 1

(10) Pulling trailers reduce the gas mileage. Suppose thatan unloaded car gets miles per gallon, and a car towing a trailer gets miles per gallon, where and are random variables with a joint density which is bivariate normal with , , , and . If a certain car gets 24 miles per gallon without a trailer, find the probability that it gets at most 15 miles per gallon with a trailer.

Answer:

(11) Let , , and be random variables with joint probability density function

Find .

Answer: .

(12) There are two gas stations at a busy intersection. The Kwick Pump and the Gas Hog. Customers arrive at the Kwick pump according to a Poisson process with rate 2 per minute and customers arrive to the Gas Hog according to a Poisson process with rate 3 per minute. Let be the number of customers arriving to the Gass Hog during the time that 4 customers arrive to the Kwick Pump. Find the expected value of .

Answer:

(13) A certain fishing vessel only keeps tuna that weigh above 50 pounds. For each tuna that they sell, they receive an amount equal to the weight of the tuna, in dollars, up to a maximum of $80. For a tuna that weighs above 80 pounds, they still receive $80. If the weight of a random tuna the vessel keeps is 50*+X,* where *X* is an exponential random variable with mean 25 pounds, find the expected amount the vessel receives for a random tuna.

Answer:

(14) Suppose that a company produces snow globes 24 hours a day, 7 days aweek. Furthermore, suppose that some of the snow globes are defective and the average number of defective snow globes produced per day is a gamma random variable with parameters 4 and . If the defective globes are produced according to a Poisson process, find the probability density function of the average number of defective snow globes after a week.

Answer: .

**Chapter 11**

(1) At a certain Doggy Weight Loss clinic, the dogs’ weights are normally distributed with mean 81 pounds and standard deviation 12 pounds. If you randomly select 12 dogs, what is the probability that the average weight of those 12 dogs is at most 80 pounds.

Answer: .

(2) is a random variable with moment generating function . Find and .

Answer:

(3) The moment generating function of a random variable is . Find for .

Answer: , , , , .

(4) The arm span of kindergarteners (in inches) is modelled by the random variable , where . Find the probability that 10 kindergarteners can encircle a 21-feet tree in circumference by touching fingertips in a circle.

Answer: .

(5) Mary drives her car every day. The error made by her car’s odometer on a random day has standard deviation 2.3 miles. Using Chebyshev’s inequality, find an upper bound for the probability that after 20 days, her odometer is off by at least 15 miles.

Answer: .

(6) For a given fishing tournament, the fish caught have lengths distributed with mean 2.3 feet and standard deviation .4 feet. If you want to be 90% sure of catching the longest fish at the tournament, what should the minimum length of the longest fish you catch be?

Answer: 4 feet

(7) For a random variable , the moment generating function is . Find and .

Answer: and .

(8) Let the moment generating function of a random variable be given by . Find .

Answer:

(9) The moment generating function for a random variable is given by . Find .

Answer:

(10) At a certain drag race, finishing times (in seconds) for Japanese cars and American cars are and respectively. If 9 American cars and 9 Japanese cars race, find the probability that the average race time of the Japanese cars is at least 2 seconds faster than that of the American cars.

Answer:

(11) Suppose that all we know about the heights of trees in a redwood forest is that their average height is 162 feet with standard deviation 22. Let be the average height of 55 trees chosen at random. Estimate

Answer: .

(12) Suppose that, on a random day in January**,** the average temperature in Duluth, MNis F with standard deviation . Give an upper bound for the probability that a random day in January in Duluth, MN is at least .

Answer: .

(13) Let be a random variable with moment generating function

for . Find the moments of .

Answer: .

(14) For a gambling game, a person wants to estimate the percentage of the times a dart player hits the bullseye. How many darts should the player throw to be 96% sure that the estimate is within .02 of the actual percentage?

Answer: .

(15) Suppose that at a certain school, all 1500 students write letters to pen pals. The number of letters each student sends is independent of the number of letters other students send. Furthermore, the numbers of letters written by students are identically distributed random variables with mean 8 and standard deviation 2.2. Find the 90th percentile of the numbers of letters written.

Answer: .

**Chapter 12**

(1) A custom auto manufacturer runs 24 hours per day and produces automobiles according to a Poisson process with rate . If over a 2-day period exactly 6 autos are produced, what is the probability thatan auto was produced every 8 hours for those 2 days?

Answer: 6!/(6^6)

(2) Construct a transition probability matrix of a Markov chain with state space in which is recurrent having period 2, {4,5,6,7} is recurrent having period 3, and {8} is aperiodic transient.

Answer:

(3) In a given town, on a given day, citizens can either go to the library (activity 1), go to the zoo (activity 2), or stay home (activity 3). Let if on the day of the year, the Shuzoku family does activity . Suppose is a Markov chain with transition probability matrix

We know the Shuzoku family went to the zoo on day 1. What is the probability they did not go to the zoo the next two days?

Answer:

(4) Let be a Markov process with state space and transition matrix

.

If is in state 1, find the expected number of transitions until getting to 2 or 3 for the first time.

Answer:

(5) To practice as a real estate agent in Tallahassee, one must get a license and keep it valid. Your license expires based on how many homes you sell. Suppose that, for an agent with a new license, the time for the license to expire is an exponential random variable with mean and the time required to renew his or her license once expired is an exponential random variable with mean . Suppose that agents let their licenses expire and then renew it. If there are 12 agents at a certain agency, find the long-run proportion of time that onl**y** 2 of those agents have valid licenses.

Answer:

(6) Geoff, Berniece, and Luis play a three-player game. Each time they play Geoff wins with probability , Berniece wins with probability and Luis wins with probability . After each play the winner gets to wear a crown, which then goes to the winner of the next game. Find the long-run proportion of the time each player gets to wear the crown.

Answer: Geoff: %45, Berniece: %.32, Luis: %.23

(7) Calls come in for a radio contest at a Poisson rate of 12 per minute. If 40 calls come in between 2:00pm and 2:04pm, what is the probability that only the last two calls arrive ~~arrived~~ in the last 10 seconds of 2:03pm?

Answer: .

(8) At a croissant factory, sometimes the croissants come out too dark. After a dark croissant is produced, out of the next 9 croissants, exactly 1 is too dark. After each normal croissant (that is, a croissant with beautiful golden color), out of the next 14 croissants, exactly 12 of them are normal.Find the long run fraction of normal croissants produced.

Answer:

(9) Each day, Quintana watches exactly one movie. She only watches action movies (genre 1), comedies (genre 2), and dramas (genre 3). Let , be a sequence of random variables with if Quintana watches a movie of genre on day . Suppose that is a Markov process with probability transition matrix

Find the long run proportion of days she watches each genre.

Answer: Action: 0.194 Comedy: 0.547 Drama: 0.259

(10) Consider the following probability transition matrix:

Find the classes of this Markov chain and determine whether each is recurrent or transient.

Answer: The only transient class is . The recurrent classes are and .

(11) A certain auto shop is an M/M/1 queuing process in which customer arrivals follow a Poisson process with rate customers per hour, and service times are exponential random variables with mean hours per customer. Find the proportion of the time there are at most 2 customers waiting in the queue.

Answer: of the time.

(12) Suppose that a bank knows that customers arrive at a Poisson rate of 8 per hour and the service time of a customer is exponential with mean hours. The bank wants to minimize the number of idle tellers. If the system is an M/M/6 queuing system (i.e., it has tellers), find the probability that there is ~~an~~ at least one idle server at any given time.

Answer: .

(13) Consider a Brownian motion with variance parameter . For what value of can we be 90% sure that the motion does not surpass in 6 seconds?

Answer: .